NOTE ON RADIUS OF CURVATURE.

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The following method of deducing the formula for the radius of curvature seems to have some pedagogical advantages. The ordinary method where infinitesimals are used leaves some doubt in the mind of the beginner as to the exactness of the result. Especially is this true when the center of curvature is defined as the intersection of two consecutive normals.

Let (x_1, y_1), (x_2, y_2) be any two points of the curve, m_1, m_2 the slope of the tangents at these points. Then

$$x+m_1y=x_1+m_1y_1,$$
$$x+m_2y=x_2+m_2y_2,$$

are the equations of the normals. Solving these equations we find that the normals intersect at the point whose coördinates are

$$x=\frac{m_2x_1+m_1m_2y_1-m_1x_2-m_1m_2y_2}{m_2-m_1}=\frac{(m_1-m_2)x_2-m_2(x_1-x_2)-m_1m_2(y_1-y_2)}{m_1-m_2}$$

$$y=\frac{(x_1-x_2)+(m_1-m_2)y_1+m_2(y_1-y_2)}{m_1-m_2};$$

$$\text{or, } x=x_2-\frac{m_2}{\dfrac{m_1-m_2}{x_1-x_2}}-\frac{m_1m_2\left(\dfrac{y_1-y_2}{x_1-x_2}\right)}{\dfrac{m_1-m_2}{x_1-x_2}},$$

$$y=y_1+\frac{1+m_2\dfrac{y_1-y_2}{x_1-x_2}}{\dfrac{m_1-m_2}{x_1-x_2}}.$$

When (x_2, y_2) approaches (x_1, y_1), we have

$$\lim(m_2)=m_1=\frac{dy_1}{dx_1};\ \lim\left(\frac{y_1-y_2}{x_1-x_2}\right)=\frac{dy_1}{dx_1};\ \lim\left(\frac{m_1-m_2}{x_1-x_2}\right)=\frac{d^2y_1}{dx_1^2}.$$

$$\text{Then } x-x_1=-\frac{dy_1}{dx_1}\left\{\frac{1+\left(\dfrac{dy_1}{dx_1}\right)^2}{\dfrac{d^2y_1}{dx_1^2}}\right\},\ y-y_1=\frac{1+\left(\dfrac{dy_1}{dx_1}\right)^2}{\dfrac{d^2y_1}{dx_1^2}}.$$

The distance of point of intersection from foot of normal is therefore

$$\rho = \frac{\left[1+\left(\frac{dy_1}{dx_1}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y_1}{dx_1^2}}.$$

The equation of a circle whose center is the point of intersection and which passes through the foot of the normal is

$$(x-x_1)^2+(y-y_1)^2=\rho^2.$$

Differentiating this twice and eliminating x, y, we find

$$\rho = \frac{\left[1+\left(\frac{dy_1}{dx_1}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y_1}{dx_1^2}}.$$

This shows that a circle having the same slope and same value of $\frac{d^2y}{dx^2}$ at its point of intersection with a given curve has its center at the limiting point of intersection of two normals.

MECHANICS.

100. Proposed by WALTER H. DRANE, Graduate Student, Harvard University; Cambridge, Mass.

A man, riding a bicycle, runs through a puddle of water and a bit of mud is thrown from the rear wheel and alights on the crown of his hat. Supposing the wheel 28 inches in diameter, that the man's head is 6 feet above ground, that the saddle is 1 foot in front of the rear wheel, and that the mud left the wheel at a point 30° from highest point of wheel, how long will it take a man to ride a mile at this rate?

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and L. R. INGERSOLL, Student Colorado College, Colorado Springs, Col.

As the particle of mud and man have the same uniform velocity forward, it is not necessary to consider such motion. The result will be the same if we regard the man at rest and the hind wheel of the bicycle revolving with the same velocity as the man is moving forward. Let O be the origin of the coördinates, D the top of the man's head, $\angle EOG=\angle OCF=\theta$. OE the tangent to the wheel at O, $CO=a=14$ inches, $HK=12$ inches, $BD=72$ inches, $g=32.16$ feet. Then $y=x\tan\theta-gx^2/2v^2\cos^2\theta$, is the equation to OD, the path of the mud.